Newton's Academy

Mathematics Part - I

Time: 2 Hours **Max. Marks: 40**

Note: i. *All* questions are compulsory.
ii. Use of a calculator is not allowed.
iii. The numbers to the right of the questions indicate full marks.
iv. In case of MCQs [Q. No. 1(A)] only the first attempt will be evaluated and will be given credit.
v. For ever MCQ, the correct alternative (A), (B), (C) or (D) with subquestion number is to be written as an answer.

Q.1. (A) For every subquestion four alternative answers are given. Choose the correct answer and write the alphabet of it: **[4]**

i. For an A.P., a = 3.5, d = 0, then t_n = _____.
(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

ii. Find the value of the determinant $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$.
(A) –1 (B) –41 (C) 41 (D) 1

iii. Which of the following quadratic equations has roots 3 and 5?
(A) $x^2 - 15x + 8 = 0$ (B) $x^2 + 8x - 15 = 0$
(C) $x^2 + 3x + 5 = 0$ (D) $x^2 - 8x + 15 = 0$

iv. There are 40 cards in a bag. Each card bears a number from 1 to 40. One card is drawn at random. What is the probability that the card bears a number which is a multiple of 5?
(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{3}$

(B) Solve the following subquestions: **[4]**

i. The sum of father's age and twice the age of his son is 70. Use the given information to form a linear equation in two variables.

ii. A die is thrown. Write sample space.

iii. Find the roots of the quadratic equation $(x + 5)(x - 4) = 0$.

iv. Find the first term and common difference for an A.P., 127, 135, 143, 151, … .

Q.2. (A) Complete and write any *two* activities from the following: **[4]**

i. Complete the following activity to find the 27th term of the following A.P.,
9, 4, –1, –6, –11, … .
Activity:
Here, a = 9, d = ☐ , n = 27
t_n = ☐ + (n – 1)d …[Formula]
∴ t_{27} = 9 + (☐ – 1) (–5)
∴ t_{27} = ☐

ii. One die is rolled. Complete the following activity, to find the probability that the number on the upper face is prime.
Activity:
S is the sample space.
S = {☐}
∴ n(S) = 6

Even A: Getting a prime number on the upper face.

A = $\{\square\}$

$\therefore$ n(A) = 3

$P(A) = \dfrac{n(A)}{\square}$ …[Formula]

$\therefore$ P(A) = $\square$

iii. Complete the following activity to find the value of x.

Activity;

$3x - y = 2$

$2x + y = 8$

$\square x = \square$

$\therefore$ $x = \dfrac{\square}{5}$

$\therefore$ $x = \square$

(B) Solve any *four* subquestions from the following: **[8]**

i. For solving the following simultaneous equations, find the values of $(x + y)$ and $(x - y)$.
$15x + 17y = 21$, $17x + 15y = 11$.

ii. Find the value of the discriminant of the quadratic equation $2y^2 - y + 2 = 0$.

iii. Find the sum of the first 21 even natural numbers.

iv. Two coins are tossed simultaneously. Find the probability of the event of getting no head.

v. Find D_x and D_y for the following simultaneous equations.
$x + 2y = -1$, $2x - 3y = 12$

Q.3. (A) Complete and write any *one* activity from the following: **[3]**

i. From three men and two women, environment committee of two persons is to be formed. To find the probabilities of the given events, complete the following activities.
Event A: There must be at least one woman member.
Event B: Committee of one man and one woman to be formed.

Activity:

Let M_1, M_2, M_3 be three men and W_1, W_2 be two women. Out of these men and women environment committee of the 2 persons is to be formed.

S = $\{M_1M_2, M_1M_3, M_2M_3, M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, \square\}$

$\therefore$ n(S) = 10

Event A: There must be at least one woman member.

A = $\{M_1W_1, M_1W_2, \square, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$

$\therefore$ n(A) = $\square$

$P(A) = \dfrac{n(A)}{n(S)}$ …[Formula]

$\therefore$ $P(A) = \dfrac{\square}{10}$

Event B: Committee of one man and one woman to be formed.

B = $\{M_1W_1, M_1W_2, M_2W_1, \square, M_3W_1, M_3W_2\}$

$\therefore$ n(B) = 6

$P(B) = \dfrac{n(B)}{n(S)}$ …[Formula]

$\therefore$ $P(B) = \frac{6}{10}$

$\therefore$ $P(B) = \frac{3}{\square}$

ii. Complete the following activity to find the roots of the quadratic equation $25x^2 + 30x + 9 = 0$ by formula method.

Activity:

$25x^2 + 30x + 9 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 25$, $b = \square$, $c = 9$

$b^2 - 4ac = (30)^2 - 4 \times 25 \times 9$

$= 900 - 900$

$= \square$

$x = \frac{\square \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-\square \pm \sqrt{0}}{2 \times 25}$

$\therefore$ $x = \frac{-30 + 0}{50}$ or $\therefore$ $x = \frac{\square - 0}{50}$

$\therefore$ $x = -\frac{30}{50}$ or $\therefore$ $x = -\frac{30}{50}$

$\therefore$ $x = -\frac{\square}{5}$ or $\therefore$ $x = -\frac{3}{5}$

(B) Attempt any *two* subquestions from the following: **[6]**

i. Solve the given equation by factorisation: $5m^2 = 22m + 15$.

ii. Solve the following equations:

$3x - 2y = \frac{5}{2}$, $\frac{1}{3}x + 3y = -\frac{4}{3}$.

iii. Length and breadth of a rectangular garden are 77 metres and 50 metres. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Find the probability of the event that it fell in the lake.

iv. A two digit number and the number with digits interchanged add up to 143. In the given number the digit in units place is 3 more than the digit in the tens place. Find the original number.

Q.4. Attempt any *two* subquestions from the following: **[8]**

i. Solve the following simultaneous equations graphically:

$x + y = 4$, $3x - 2y = 7$.

ii. A train travels 240 km with uniform speed. If the speed of the train is increased by 12 km/h, it takes one hour less to cover the same distance. Find the initial speed of the train.

iii. If the sum of the first p terms of an A.P. is equal to the sum of first q terms, then show that the sum of its first (p + q) terms is zero ($p \neq q$).

Q.5. Solve the following subquestions: (Any *one*) **[3]**

i. The measures of the angles of a quadrilateral are in A.P. The measure of largest angle is twice the smallest. Find the measures of all angles of the quadrilateral.
[Assume measures of angles as a°, (a + d)°, (a + 2d)°, (a + 3d)°,
(where $a < a + d < a + 2d < a + 3d$)]

ii. The product of two numbers is 352 and their mean is 19. Find the numbers.